University of Manitoba

Department of Electrical and Computer Engineering

ECE 4390 Engineering Computations IV

Laboratory 5: The Finite-Difference Time-Domain Method for EM

Tuesday, November 16, 2010 **Due Date: Tuesday, November 30, 2010**

Part I: One-Dimensional Plane Waves

1) Starting from Maxwell's equations and assuming that the fields vary only in the x-dierection show that the governing equations for the $E_y(x, t)$ and $H_z(x, t)$ field components in a general lossy medium characterized by $e = \varepsilon^{-1}$, $m = \mu^{-1}$ and conductivity σ are

$$\begin{cases} \partial_t E_y + e\partial_x H_z + \sigma e E_y = 0 \\ \partial_t H_z + m\partial_x E_y = 0 \end{cases}.$$

2) Derive an appropriate explicit update scheme based on Yee's interlaced version of the Leap-Frog finite-difference method for the above equations. Use the following discretization:

$$E_i^n = E_y(i\Delta x, n\Delta t)\,,\; H_{i+1/2}^{n+1/2} = H_z((i+1/2)\Delta x, (n+1/2)\Delta t)\,.$$

3) Write a program to solve the following problem shown in Figure 1. The boundaries of the problem are defined to be the 3 [m] region between $x = 0$ and $x = 300$ [cm]. Use a discretization of $\Delta x = 1$ [cm]. A dielectric slab having $\varepsilon_r = 3$ exists as shown and the left-most 50 [cm] of the region is a lossy medium having $\sigma = 0.01$ [S/m]. Write the up-date equations for each region as well as the boundary conditions.

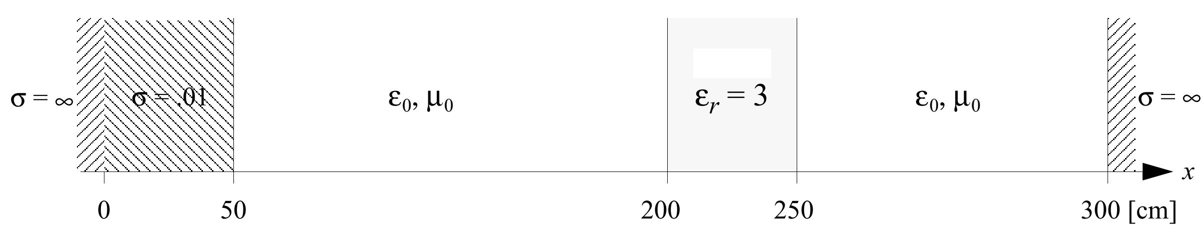


Figure 1. Dielectric slab and perfectly conducting boundaries.

4) As initial conditions use the Gaussian waveform shown in Figure 2. for the electric field. Determine initial conditions on the magnetic field which represent an initial wave propagating solely to the right. What is the maximum Δt for stability? Plot the electric and magnetic fields at times n = {25, 50, 75, 100, 125, 150, 175, 200, 300, 400, 500} across the whole x axis.

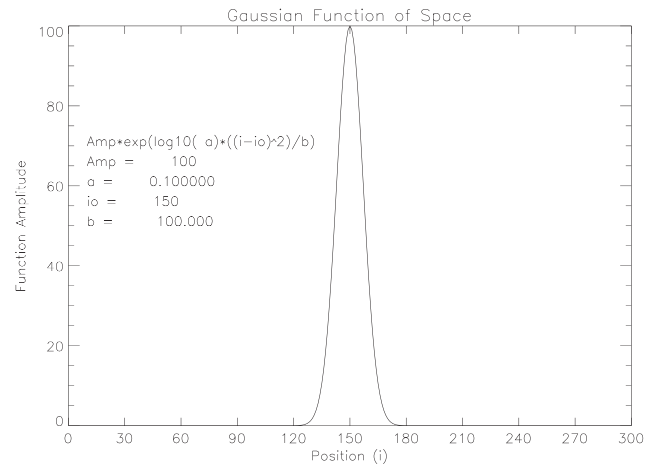
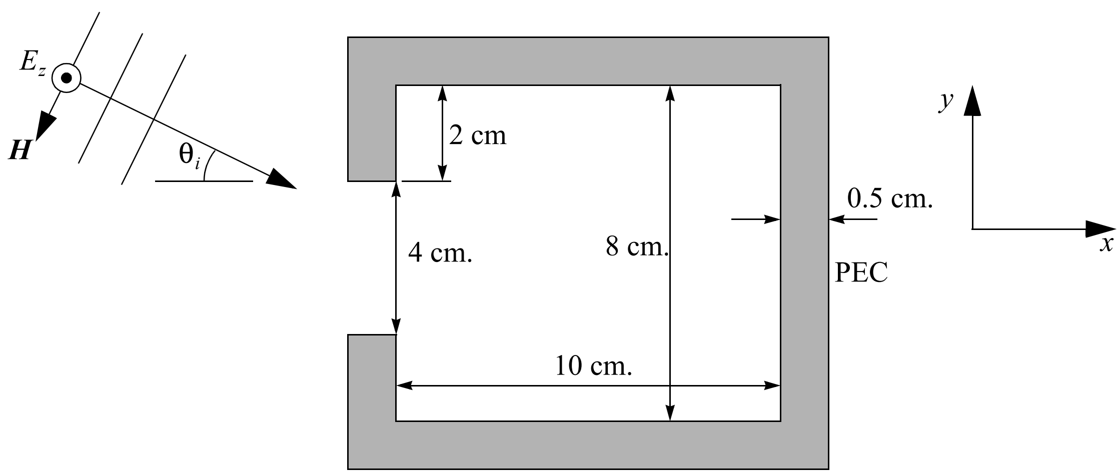


Figure 2. Initial conditions on electric field: $E_i^0 = 100\, exp\left[-\left(\frac{(i-150)^2}{100}\right)\right]$

Part II: Two-Dimensionsal TM waves

Write a 2-D FDTD program to solve the shielding problem shown in the figure. Describe the algorithm and the set-up for the problem fully.



The incident plane-wave is polarized as transverse-magnetic to the z-direction (*i.e.*, only an E_z component of the electric field exists and there is no H_z component of the magnetic field). Use the scattered-field formulation of FDTD and plot the shielding effectiveness as a function of frequency for 1 MHz to 10 GHz.

The shielding effectiveness is defined as:

$$SE = -20\log\left(\frac{\max \tilde{E}_z \text{ inside shield}}{\text{value of } \tilde{E}_z \text{ without shield}}\right)$$

where $\tilde{E}_z$ is the frequency-domain electric field. Do this for various angles of incidence from $\theta_i = 0$ to $\theta_i = \pi$. Use Mur's second-order absorbing boundary conditions you like to terminate the grid.